

# The $CP(N-1)$ Affine Gauge Theory in the Dynamical Space-time

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## Abstract

An attempt to build quantum theory of field (extended) objects without a priori space-time geometry has been represented. Space-time coordinates are replaced by the intrinsic coordinates in the tangent fibre bundle over complex projective Hilbert state space  $CP(N-1)$ . The fate of quantum system modeled by the generalized coherent states is rooted in this manifold. Dynamical (state-dependent) space-time arises only at the stage of the quantum “yes/no” measurement. The quantum measurement of the gauge “field shell” of the generalized coherent state is described in terms of the affine parallel transport of the local dynamical variables in  $CP(N-1)$ .

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## 1 Introduction

The deep disagreement between general relativity and quantum theory is well known [1]. In this work I would like represent the model of quantum geometry, intended to reach the desirable “peaceful coexistence” between these theories. The proposed scheme is inherently based on the notion of the relative quantum amplitudes. Formally, I deal with “classical” non-linear field theory developed over the complex projective Hilbert space  $CP(N-1)$ . It is worth while to emphasize, that my approach

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[2, 3, 4, 5, 6] quite differs from number of works using  $CP(N - 1)$ ; see for example [7, 8, 9].

The sketch of the proposed scheme is as follows:

a). I use the realization of the  $G = SU(N)$  acting on the states  $|S\rangle \in \mathcal{H} = C^N$  in terms of local dynamical variables (LDV's) represented by the tangent vectors to  $CP(N - 1)$  (the operators of differentiation).

b). Quantum measurement realized as a perturbation of the generalized coherent quantum state (GCS).

c). The self-identification of quantum system is realized by the affine parallel transport of its local dynamical variables, agrees with Fubini-Study metric.

d). Variation principle applied to the local Hamiltonian vector field leads to quasi-linear PDE field equations for the “field shell” of the GCS. This “field shell” represents some “quantum potential” of the model extended “particle” corresponding GCS.

e). The “yes/no” measuring process formulated as a detection of this extended particle serving for the establishment of the local state-dependent space-time structure.

This approach leads to some conclusion concerning so-called measurement problem. It is convenient to refer to the encyclopedic book of R. Penrose [1].

1. Projective postulate and null-measurement.

The so-called null-measurement (see paragraph 22.7, [1]) is in fact a non-relevant construction, since the conclusion that “we know that photon is in state  $|\rho\rangle$  even though it has not interacted with the detector (in the transmission channel  $|\tau\rangle$ -P.L.) at all”, is based on the explicit belief that the photon already has passed splitter. But photon might be simply absorbed even before this splitter. Therefore, strictly speaking, we do not have reliable information without a detection in the corresponding channel. This example shows that if one has left some gap between two successive quantum states, the application of the projective postulate (if no  $|\tau\rangle$ , then  $|\rho\rangle$ ) is meaningless.

In the framework of my model the projection acts continuously and locally along  $CP(N - 1)$  trajectory of GCS onto the corresponding tangent spaces, since it is the covariant differentiation of vector fields representing local dynamical variables (LDV's) on  $CP(N - 1)$ .

2. Deformation of GCS during interaction used for measurement.

Let me discuss dynamics of Schrödinger's lump during measurement (see paragraph 30.10, [1]). This construction is a humane version of the Schrödinger's cat. In distinguish with so complicated system as poisoned cat, and indefinite displaced lump of matter, we would like discuss the deformation of GCS which is theoretically analyzable.

First of all I should note that the assumption that “the energy in each case is the same” may be correct only approximately, say, in the case of adiabatic “kicking” of the lump. The finite time of transition unavoidably leads to the acceleration of the lump of matter, to the deformation of its quantum state [10, 11], and to the shift of mass-energy. Hence, the superposition state is not stationary (beating) and, therefore, this is useless for our decision about real interaction process of the photon and splitter (as well as in the original “comic example” of Schrödinger’s cat demonstrating incompleteness of the wave function description of an nuclear decay [12]).

In the framework of my model, the GCS of the lump is “kicked” in the first approximation by the coset transformations of  $SU(N)$  group. The coefficient functions of the  $SU(N)$  generators obey some quasi-linear relativistic field equations in the local dynamical space-time [2, 3, 4].

3. The difference of the masses of the original and the displaced lumps leads to different time-like Killing vectors (if any) in the vicinities of two lumps. This is an obstacle to write Schrödinger equations for superposed wave function. But, who does need it? This is rather a privilege than a defect, since one has a natural decoherence mechanism.

In the framework of my model one has state-dependent space-times arising as specific cross-section of the tangent fibre bundle over  $CP(N-1)$ . Linear superposition has a sense only in dynamical space-time in the quantum system (setup, lump) under the condition of *physical integrity*. In general, the formulation of the physical integrity is a difficult problem; in my model the GCS expresses this property. This leads to the dynamics in the tangent fibre bundle over  $CP(N-1)$ . All thought compounds of free independent systems are trivial since they live in the tensor product of the state spaces.

Below will be introduced some fundamental notions of my construction.

## 2 Action states with entire number $N$ of $\hbar$

The masses of known “elementary” particles  $m_J$  are in the fundamental de Broglie relation to corresponding internal frequencies  $\omega_J$ :

$$\frac{\omega_J}{m_J} = \frac{c^2}{\hbar}. \quad (2.1)$$

If one treat the  $U = c^2$  as the cosmic potential, then arise the natural question about the micro-selective mechanism capable produce very specific spectrum of fre-

quencies. In the ordinary quantization scheme it is assumed that the oscillator is really some fundamental entity. But the spectrum of oscillator is equidistant and unbounded whereas the mass-spectrum of “elementary” particles does not. Furthermore, the classical soliton-like solution cannot be decomposed into harmonic waves, hence quantum solitons are not a compound of quantum oscillators. I try to find a dispersion law  $\Omega(P)$  (initially in the form  $\Omega(X)$ ) as a solution of the non-linear field equations.

There are some additional reasons for the modification of the “second quantization” procedure.

*First.* In the second quantization method one has formally given particles whose properties are defined by some commutation relations between creation-annihilation operators. Note, that the commutation relations are only the simplest consequence of the curvature of the dynamical group manifold in the vicinity of the group’s unit (in algebra). Dynamical processes require, however, finite group transformations and, hence, the global group structure. The main technical idea is to use vector fields over group manifold instead of indefinite Dirac’s q-numbers. This scheme therefore looking for the dynamical nature of the creation and annihilation processes of quantum particles.

*Second.* The quantum particles (energy bundles) should gravitate. Hence, strictly speaking, their behavior cannot be described as a linear superposition. Therefore the ordinary second quantization method (creation-annihilation of free particles) is merely a good approximate scheme due to the weakness of gravity. Thereby the creation and annihilation of particles are time consuming dynamical non-linear processes. So, linear operators of creation and annihilation (in Dirac sense) do exist as approximate quantities.

*Third.* Nobody knows how arise a quant of energy (quantum particle). Definitely, there is an energy quantization but the dynamical nature of this process is unknown. Avoiding the vacuum stability problem, its self-energy, etc., we primary quantize, however, the action, not energy. The relative (local) vacuum of some problem is not the state with minimal energy, it is a state with an extremal of some action functional.

#### POSTULATE 1.

*We assume that there are elementary quantum states (EQS)  $|\hbar a\rangle$ ,  $a = 0, 1, \dots$  of abstract Planck’s oscillator whose states correspond to the quantum motions with given number of Planck’s action quanta.*

Thereby only action subject to primary quantization but the quantization of dynamical variables such as energy, spin, etc., postponed to dynamical stage. Presumably there are some non-linear field equations describing energy (frequency) distribution, whose soliton-like solution provides the quantization of the dynamical variables

but their field carriers - “field shell” are smeared in dynamical space-time. Therefore, quantum “particles”, and, hence, their numbers should arise as some countable solutions of non-linear wave equations. In order to establish acceptable field equation capable intrinsically to describe all possible degrees of freedom defreezing under intensive interaction, we should build some *universal ambient Hilbert state space*  $\mathcal{H}$ . We will use *the universality of the action* whose variation capable generate any dynamical variable. Vectors of *action state space*  $\mathcal{H}$  we will call *action amplitude* (AA). Some of them will be EQS’s of motion corresponding to entire numbers of Planck’s quanta  $|\hbar a\rangle$ . Since the action in itself does not create gravity, it is legible to create the linear superposition of  $|\hbar a\rangle = (a!)^{-1/2}(\hat{\eta}^+)^a|0\rangle$  constituting  $SU(\infty)$  multiplete of the Planck’s action quanta operator  $\hat{S} = \hbar\hat{\eta}^+\hat{\eta}$  with the spectrum  $S_a = \hbar a$  in the separable Hilbert space  $\mathcal{H}$ . The standard basis  $\{|\hbar a\rangle\}_0^\infty$  will be used with the ‘principle’ quantum number  $a = 0, 1, 2, \dots$  assigned by Planck’s quanta counting. Generally (AA) are their coherent superposition

$$|G\rangle = \sum_{a=0}^{\infty} g^a |\hbar a\rangle. \quad (2.2)$$

may represented of the ground state - “vacuum” of some quantum system. In order to avoid the misleading reminiscence about Schrödinger state vector, I use  $|G\rangle$ ,  $|S\rangle$  instead of  $|\Psi\rangle$ . In fact only finite, say,  $N$  EQM may be involved. Then one may restrict  $CP(\infty)$  QPS to finite dimensional  $CP(N-1)$ . Hereafter I will use the indices as follows:  $0 \leq a \leq N$ , and  $1 \leq i, k, m, n, s \leq N-1$ .

### 3 Quantum analog of force and $SU(N)$ factorization

Since any ray AA has isotropy group  $H = U(1) \times U(N)$ , only coset transformations  $G/H = SU(N)/S[U(1) \times U(N-1)] = CP(N-1)$  effectively act in  $\mathcal{H}$ . Therefore the ray representation of  $SU(N)$  in  $C^N$  and, in particular, the embedding of  $H$  and  $G/H$  in  $G$ , require a state-dependent parametrization. Hence, there is a diffeomorphism between space of the rays marked by the local coordinates in the map  $U_j : \{|G\rangle, |g^j| \neq 0\}, j > 0$

$$\pi_{(j)}^i = \left\{ \begin{array}{ll} \frac{g^i}{g^j} & \text{if } 1 \leq i < j \\ \frac{g^{i+1}}{g^j} & \text{if } j \leq i < N-1 \end{array} \right\} \quad (3.1)$$

and the group manifold of the coset transformations  $G/H = SU(N)/S[U(1) \times U(N-1)] = CP(N-1)$ . This diffeomorphism is provided by the coefficient functions  $\Phi_\alpha^i$

the local generators (see below and [10, 11]). The choice of the map  $U_j$  means, that the comparison of quantum amplitudes refers to the amplitude with the action  $\hbar j$ . The breakdown of  $SU(N)$  symmetry on each AA to the isotropy group  $H = U(1) \times U(N-1)$  contracts full dynamics down to  $CP(N-1)$ . The physical interpretation of these transformations is given by the

POSTULATE 2.

*Super-equivalence principle: the unitary transformations of the AA may be identified with the physical unitary fields. The coset transformation  $G/H = SU(N)/S[U(1) \times U(N-1)] = CP(N-1)$  is the quantum analog of classical force: its action is equivalent to some physically distinguishable variation of GCS in  $CP(N-1)$ .*

The  $CP(N-1)$  manifold takes the place of “classical phase space” [15], since its points, corresponding to the GCS, are most close to classical states of motion. Two interpretations may be given for the points of  $CP(N-1)$ . One of them is the “Schrödinger’s lump” [1] and the second one is the analog of the Stern-Gerlach “filter’s orientations” discussed by Fivel [16]. The root content of their physical interpretations is that one has a *macroscopic (i.e. space-time) discriminator* of two quantum states. As such, they may be used as “yes/no” states of some two-level detector. We will use the “Schrödinger’s lump” interpretation. Let us assume that GCS described by local coordinates  $(\pi^1, \dots, \pi^{N-1})$  correspond to the original lump, and the coordinates  $(\pi^1 + \delta\pi^1, \dots, \pi^{N-1} + \delta\pi^{N-1})$  correspond to the displaced lump. Such hidden coordinates of the lump gives a firm geometric tool for the description of quantum dynamics during interaction used for the measuring process.

Then the question that I now want to rise is as following: *what “classical field”, i.e field in space-time, correspond to the transition from the original to the displaced lump?* In other words we would like find the “field shell” of the lump, its space-time shape and its dynamics. The lump’s perturbations will be represented by the “geometric bosons” [17] whose frequencies are not a priori given, but which defined by some field equations which should established due to a new variation problem. Before its formulation, we ought to use in fact a sophisticated differential geometric construction in order to avoid the clash between quantum mechanics and general relativity [1].

I will assume that all “vacua” solutions belong to single separable *projective Hilbert space*  $CP(N-1)$ . The vacuum represented by GCS is merely the stationary point of some action functional, not solution with the minimal energy. Energy will be associated with tangent vector field to  $CP(N-1)$  giving velocity of the action variation in respect with the notion of the Newton-Stueckelberg-Horwitz-Piron (NSHP) time [18]. Dynamical (state-dependent) space-time will be built at any GCS and, particularly, at the vacuum of some “classical” problem (see below). Therefore Minkowskian

space-time is functionally local (state-dependent) in  $CP(N-1)$  and the space-time motion dictated by the field equations connected with two infinitesimally close GCS. The connection between these local space-times may be physically established by the measurement given in terms of geometry of the base manifold  $CP(N-1)$ . It seems to be like the Everett's idea about "parallel worlds", but has of course different physical sense. Now we are evidences of the Multiverse (omnium) concept [1, 19]. I think there is only one Universe but there exists continuum of dynamical space-times each of them related to one point of the quantum phase space  $CP(N-1)$ . The standard approach, identifying Universe with space-time, is too strong assumption from this point of view.

## 4 LDV's and tangent fibre bundles

The state space  $\mathcal{H}$  of the field configurations with finite action quanta is a stationary construction. We introduce dynamics *by the velocities of the GCS variation* representing some "elementary excitations" (quantum particles). Their dynamics is specified by the Hamiltonian, giving time variation velocities of the action quantum numbers in different directions of the tangent Hilbert space  $T_{(\pi^1, \dots, \pi^{N-1})}CP(N-1)$  where takes place the ordinary linear quantum scheme. The temp of the action variation gives the energy of the "particles".

The local dynamical variables corresponding internal symmetries of the GCS and their breakdown should be expressed now in terms of the local coordinates  $\pi^k$ . The Fubini-Study metric

$$G_{ik^*} = [(1 + \sum |\pi^s|^2)\delta_{ik} - \pi^{i*}\pi^k](1 + \sum |\pi^s|^2)^{-2} \quad (4.1)$$

and the affine connection

$$\Gamma_{mn}^i = \frac{1}{2}G^{ip^*}(\frac{\partial G_{mp^*}}{\partial \pi^n} + \frac{\partial G_{p^*n}}{\partial \pi^m}) = -\frac{\delta_m^i \pi^{n*} + \delta_n^i \pi^{m*}}{1 + \sum |\pi^s|^2} \quad (4.2)$$

in these coordinates will be used. Hence the internal dynamical variables and their norms should be state-dependent, i.e. local in the state space [10, 11]. These local dynamical variables realize a non-linear representation of the unitary global  $SU(N)$  group in the Hilbert state space  $C^N$ . Namely,  $N^2 - 1$  generators of  $G = SU(N)$  may be divided in accordance with Cartan decomposition:  $[B, B] \in H, [B, H] \in B, [B, B] \in H$ . The  $(N-1)^2$  generators

$$\Phi_h^i \frac{\partial}{\partial \pi^i} + c.c. \in H, \quad 1 \leq h \leq (N-1)^2 \quad (4.3)$$

of the isotropy group  $H = U(1) \times U(N - 1)$  of the ray (Cartan sub-algebra) and  $2(N - 1)$  generators

$$\Phi_b^i \frac{\partial}{\partial \pi^i} + c.c. \in B, \quad 1 \leq b \leq 2(N - 1) \quad (4.4)$$

are the coset  $G/H = SU(N)/S[U(1) \times U(N - 1)]$  generators realizing the breakdown of the  $G = SU(N)$  symmetry of the GCS. Furthermore,  $(N - 1)^2$  generators of the Cartan sub-algebra may be divided into the two sets of operators:  $1 \leq c \leq N - 1$  (where  $N - 1$  is the rank of  $AlgSU(N)$ ) Abelian operators, and  $1 \leq q \leq (N - 1)(N - 2)$  non-Abelian operators corresponding to the non-commutative part of the Cartan sub-algebra of the isotropy (gauge) group. Here  $\Phi_\sigma^i$ ,  $1 \leq \sigma \leq N^2 - 1$  are the coefficient functions of the generators of the non-linear  $SU(N)$  realization. They give the infinitesimal shift of  $i$ -component of the coherent state driven by the  $\sigma$ -component of the unitary multipole field rotating the generators of  $AlgSU(N)$  and they are defined as follows:

$$\Phi_\sigma^i = \lim_{\epsilon \rightarrow 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon\lambda_\sigma)]_m^i g^m}{[\exp(i\epsilon\lambda_\sigma)]_m^j g^m} - \frac{g^i}{g^j} \right\} = \lim_{\epsilon \rightarrow 0} \epsilon^{-1} \{ \pi^i(\epsilon\lambda_\sigma) - \pi^i \}, \quad (4.5)$$

[2]. Then the sum of  $N^2 - 1$  the energies of the ‘elementary systems’ (particle plus fields) is equal to the excitation energy of the GCS, and the local Hamiltonian  $\vec{H}$  is linear against the partial derivatives  $\frac{\partial}{\partial \pi^i} = \frac{1}{2}(\frac{\partial}{\partial \Re \pi^i} - i\frac{\partial}{\partial \Im \pi^i})$  and  $\frac{\partial}{\partial \pi^{*i}} = \frac{1}{2}(\frac{\partial}{\partial \Re \pi^i} + i\frac{\partial}{\partial \Im \pi^i})$ , i.e. it is the tangent vector to  $CP(N - 1)$

$$\vec{H} = \vec{T}_c + \vec{T}_q + \vec{V}_b = \hbar\Omega^c \Phi_c^i \frac{\partial}{\partial \pi^i} + \hbar\Omega^q \Phi_q^i \frac{\partial}{\partial \pi^i} + \hbar\Omega^b \Phi_b^i \frac{\partial}{\partial \pi^i} + c.c.. \quad (4.6)$$

The characteristic equations for the PDE  $\vec{H}|E\rangle = E|E\rangle$  give the parametric representations of their solutions in  $CP(N - 1)$ . The parameter  $\tau$  in these equations I will identify with ‘universal time of evolution’ of Newton-Stueckelberg-Horwitz-Piron-(NSHP) [18]. This time is the measure of the GCS variation, i.e. it is a state-dependent measure of the distance in  $CP(N - 1)$  (an evolution trajectory length in the Fubini-Study metric) expressed in time units. The energy quantization will be discussed elsewhere.

## 5 Lorentz transformations and dynamical space-time

The Einstein’s analysis of the Galileo-Newtonian kinematics in an inertial reference frame, based on the classical Maxwell’s electromagnetic field theory, led us to a new



relativistic kinematics [20]. Unfortunately, similar analysis based on the quantum theory is in a very preliminary state [1, 21]. The continuation of such a work is necessary.

It is clear that the *coincidence* of the “arrow” with some number on the “limb” is in fact the coincidence of the two space-time points. But one has in the quantum area literally neither “arrow” nor the “limb”; some “clouds” or “field shell” one has instead. Thereby the uncertainty principle puts the limit for the exact coincidence of two events. Therefore, in comparison with us, Einstein had two privileges: he had intuitively clear classical measuring devices (clocks, scales, rods, etc.) and the intuitively clear spatial coincidence of two “points”, say, the end of a rod and the end of the scale. Without these ingredients it is difficult to image the measurement process and even the space-time notion itself. Generally, space-time coordinates lose direct physical sense even in the framework of general relativity [22]. Quantum theory poses a new problem concerning operational sense of the microscopic invariance of the space-time scale. Indeed, all abstract (notional) tools of macroscopic laboratory (clocks, scales, rods, etc.) one should change for the microscopic ones. Note, Bohr’s proposal about “classical apparatus” is unacceptable since it is inconsistent. We should construct now the space-time notion in the internal quantum terms.

The notion of physical space is based on the abstraction of separation between two material points assuming that they may be as far as we need. This separation may be measured by some hard scale. The “hard scale” in fact means the “the same” or identical scale, i.e. the scale with invariant length relative some transformation group. But in quantum theory it is inconsistent (or it is at least questionable) to use a priori space-time symmetries. Similar arguments is applicable to time separation because of the specific problem of the “time-of-arrival” [23]. Generally speaking, *space-time separation is state-dependent*. In such situation, one should decide on what is the criterion of identity is physically acceptable in our case. If, say, electrons used for the measurement of separation between a source  $S_1$  and a detector  $D_1$ , than the most reasonable to use the criterion of the “same electron” (emitted from  $S_1$  and detected in  $D_1$ ). All quantum electrons are of course identical but there are momentum and spin which distinguish one electron from another. But in general this criterion is not so good as we need since we cannot be sure that the electron detected in  $D_2$  is same as in  $D_1$ , or even that this has some causal connection with the previous stage of the measurement. There is at least one reason for this verdict: the detection of some accidental electron, e.g. due to a quantum fluctuations, etc. Nevertheless, in the bubble chamber one may be sure that whole visible trace belongs to “same electron”. Therefore, if interaction is not so drastic or, if one takes into account all possible decay channels of an unitary multiplete, we could formulate the criterion of identity. Let

me to formulate this criterion previously with promise to decode all components of the statement: *the local Hamiltonian should be parallel transported during a “smooth” evolution*. I introduce the concept of “dynamical space-time” as a new construction capable to detect the coincidences of the qubit components in the formal two-level “detector” which is a part of the full quantum configuration (setup modeled by the GCS). The “extraction” of this “detector” is of course more or less a free choice of an observer. It is important only that the chosen LDV should be invariantly connected with the qubit coherent state in respect with a one of the points of the LDV spectrum. I will assume that the spectrum of the LDV is known even if it is really problematic like, for example, in the PDE eigen-problem  $\vec{H}|E\rangle = E|E\rangle$  mentioned above.

## 5.1 Embedding “Hilbert (quantum) dynamics” in space-time

If we would like to have some embedding of the “Hilbert (quantum) dynamics” in space-time we should formalize the quantum observation (or measurement of some internal dynamical variable).

Mentioned above diffeomorphism between rays of  $CP(N-1)$  and  $SU(N)$  generators will be realized in terms of the local  $SL(2, C)$  action onto the qubit states space  $C^2$  as follows.

The basis of these spaces form two vectors: the normal vector  $|N\rangle$  to the “vacuum landscape”  $CP(N-1)$  corresponding to eigenvalue  $\lambda_D$  of measuring dynamical variable  $\hat{D}$  and the tangent vector  $|T\rangle$ , generated by the coset generators of  $G/H$ . The last ones describing the interaction used for the measurement process. It is important to understand that the measurement i.e. comparison of the expected qubit spinor  $(\alpha_0, \beta_0)$  at and measured qubit spinor  $(\alpha_1, \beta_1)$  pave the way to embedding Hilbert space dynamics into the local dynamical space-time. This is the replacement of the notorious “arrow” of the measuring device, namely: one has two-level system (logical spin 1/2 [10]) created by the quantum question-unitary projector onto one of the two states  $|N\rangle, |T\rangle$ . Their coherent states are given by the qubit spinors  $(\alpha, \beta)$  being connected with infinitesimal  $SL(2, C)$  transformations give rise to the variation of the space-time coordinates generated by local infinitesimal Lorentz transformations. Why we can do this conclusion?

Causal classical events lie (in good approximation) on a light cone which is invariant relative the Lorentz group. On the other hand the formal “Lorentz spin transformations matrix” transform the spinor of the quantum question being applied to measurement of some LDV helping us to detect some event. The classical detection of an event is based on the coincidence of the two spinors one of which corresponds to the expectation value and the second to detecting value of LDV. This is possible

only under the tuning of orientation by rotation and the tuning of velocity by acceleration. Therefore we should identify “Lorentz spin transformations matrix” of the qubit spinors with Lorentz transformation of classical inertial frame.

The specific components of LDV’s (see below) take the place of these entities. But now LDV is vector field defined over  $CP(N - 1)$  and the comparison of LDV at different setups (initial and perturbed due to interaction used for measurement) require some procedure of the *self-identification*. It is impossible to compare expected and measured LDV “directly” (decoherence due to  $CP(N - 1)$  geometry [5]). The affine parallel transport is quite acceptable for this aim. The parallel transport forms the condition for the coefficient functions of the LDV leading to the nonlinear field equations in the local dynamical space-time.

## 5.2 Differential geometry of the measuring procedure

The measurement, i.e. attributing a number to some dynamical variable or observable has in physics subjective as well as objective sense. Namely: the numeric value of some observable depends as a rule on a setup (the character of motion of laboratory, type of the measuring device, field strength, etc.). However the relationships between numeric values of dynamical variables and numeric characteristics of laboratory motion, field strength, etc., should be formulated as invariant, since they reflect the objective character of the physical interaction used in the measurement process. The numbers obtained due to the measurements carry information which does not exist a priori, i.e. before the measurement process. But the information comprised of subjective as well as objective invariant part reflects the physics of interaction. The last is one of the main topics of QFT. Since each measurement reducible (even if it is unconscious) to the answer of the question “yes” or “no”, it is possible to introduce formally a quantum dynamical variable “logical spin  $1/2$ ” [10] whose coherent states represent the quantum bit of information “qubit”.

### POSTULATE 3

*We assume that the invariant i.e. physically essential part of information represented by the coherent states of the “logical spin  $1/2$ ” is related to the space-time structure.*

Such assumption is based on the observation that on one side the space-time is the manifold of points modeling different physical systems (stars, atoms, electrons, etc.) artificially depleted of all physical characteristics (material points without reference to masses). In principle arbitrary local coordinates may be attributed to these points. But as we know from general relativity the metric structure depends on the matter distribution and the zero approximation of the metric tensor  $g_{\mu\nu} = \eta_{\mu\nu} + \dots$  gives the

Lorentz invariant interval [22]. On the other hand the spinor structure of the Lorentz transformations represents the transformations of the coherent states of the “logical spin 1/2” or “qubit”. Thereby we can assume the measurement of the quantum dynamical variables expressed by the “qubit” spinor “creates” the local space-time coordinates. We will formulate non-linear field equations in this local space-time due to a variational principle referring to the generator of the quantum state deformation.

The internal hidden dynamics of the quantum configuration given by GCS should be somehow reflected in physical space-time. Therefore we should solve the “inverse representation problem”: to find locally unitary representation of dynamical group  $SU(N)$  in the dynamical space-time where acts the induced realization of the coherence group  $SU(2)$  of the qubit spinor [2, 3]. Its components subjected to the “Lorentz spin matrix transformations” [14]. We should build the local spinor basis invariantly related to the ground states manifold  $CP(N - 1)$ . First of all we have to have the local reference frame (LRF) as some analog of the “representation” of  $SU(N)$ . Each LRF and, hence,  $SU(N)$  “representation” may be marked by the local coordinates (3.1) of the “vacuum landscape”. Now we should almost literally repeat differential geometry of a smooth manifold embedded in flat ambient Hilbert space  $\mathcal{H} = C^N$ . The geometry of this smooth manifold is the projective Hilbert space equipped with the Fubini-Study metric (4.1) and with the affine connection (4.2).

In order to express the measurement of the “particle’s field” in the geometrically intrinsic terms, I assume that GCS is expressed in the local coordinates

$$|G(\pi^1, \dots, \pi^{N-1})\rangle = (g^0(\pi^1, \dots, \pi^{N-1}), g^1(\pi^1, \dots, \pi^{N-1}), \dots, g^{N-1}(\pi^1, \dots, \pi^{N-1}))^T, \quad (5.1)$$

where  $\sum_{a=0}^N |g^a|^2 = R^2$ , and, hence,

$$g^0(\pi^1, \dots, \pi^{N-1}) = \frac{R^2}{\sqrt{R^2 + \sum_{s=1}^{N-1} |\pi^s|^2}}, \quad (5.2)$$

and for  $1 \leq i \leq N - 1$  one has

$$g^i(\pi^1, \dots, \pi^{N-1}) = \frac{R\pi^i}{\sqrt{R^2 + \sum_{s=1}^{N-1} |\pi^s|^2}}, \quad (5.3)$$

i.e.  $CP(N - 1)$  will be embedded in the Hilbert space of Planck’s quanta  $\mathcal{H} = C^N$ .

Then the velocity of ground state evolution relative NSHP time is given by the formula

$$|H\rangle = \frac{d|G\rangle}{d\tau} = \frac{\partial g^a}{\partial \pi^i} \frac{d\pi^i}{d\tau} |a\hbar\rangle = |T_i\rangle \frac{d\pi^i}{d\tau} = H^i |T_i\rangle, \quad (5.4)$$

is the tangent vector to the evolution curve  $\pi^i = \pi^i(\tau)$ , where

$$|T_i \rangle = \frac{\partial g^a}{\partial \pi^i} |a\hbar \rangle = T_i^a |a\hbar \rangle. \quad (5.5)$$

Then the “acceleration” is as follows

$$\begin{aligned} |A \rangle = \frac{d^2 |G \rangle}{d\tau^2} = |g_{ik} \rangle \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} + |T_i \rangle \frac{d^2 \pi^i}{d\tau^2} = |N_{ik} \rangle \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} \\ + \left( \frac{d^2 \pi^s}{d\tau^2} + \Gamma_{ik}^s \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} \right) |T_s \rangle, \end{aligned} \quad (5.6)$$

where

$$|g_{ik} \rangle = \frac{\partial^2 g^a}{\partial \pi^i \partial \pi^k} |a\hbar \rangle = |N_{ik} \rangle + \Gamma_{ik}^s |T_s \rangle \quad (5.7)$$

and the state

$$|N \rangle = N^a |a\hbar \rangle = \left( \frac{\partial^2 g^a}{\partial \pi^i \partial \pi^k} - \Gamma_{ik}^s \frac{\partial g^a}{\partial \pi^s} \right) \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} |a\hbar \rangle \quad (5.8)$$

is the normal to the “hypersurface” of the ground states. Then the minimization of this “acceleration” under the transition from point  $\tau$  to  $\tau + d\tau$  may be achieved by the annihilation of the tangential component

$$\left( \frac{d^2 \pi^s}{d\tau^2} + \Gamma_{ik}^s \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} \right) |T_s \rangle = 0 \quad (5.9)$$

i.e. under the condition of the affine parallel transport of the Hamiltonian vector field

$$dH^s + \Gamma_{ik}^s H^i d\pi^k = 0. \quad (5.10)$$

The Gauss-Codazzi equations

$$\begin{aligned} \frac{\partial N^a}{\partial \pi^i} &= B_i^s T_s^a \\ \frac{\partial T_k^a}{\partial \pi^i} - \Gamma_{ik}^s T_s^a &= B_{ik} N^a \end{aligned} \quad (5.11)$$

I used here instead of the anthropic principle [1, 19]. These give us dynamics of the vacuum (normal) vector and the tangent vectors, i.e. one has the LRF dynamics modeling the “moving representation” or moving quantum setup

$$\frac{dN^a}{d\tau} = \frac{\partial N^a}{\partial \pi^i} \frac{d\pi^i}{d\tau} + c.c. = B_i^s T_s^a \frac{d\pi^i}{d\tau} + c.c. = B_i^s T_s^a H^i + c.c.;$$

$$\begin{aligned}\frac{dT_k^a}{d\tau} &= \frac{\partial T_k^a}{\partial \pi^i} \frac{d\pi^i}{d\tau} + c.c. = (B_{ik}N^a + \Gamma_{ik}^s T_s^a) \frac{d\pi^i}{d\tau} + c.c. \\ &= (B_{ik}N^a + \Gamma_{ik}^s T_s^a) H^i + c.c.\end{aligned}\quad (5.12)$$

Please, remember that  $0 \leq a \leq N$ , but  $1 \leq i, k, m, n, s \leq N-1$ . The tensor  $B_{ik}$  of the second quadratic form of the ground states “hypersurface” is as follows:

$$B_{ik} = \langle N | \frac{\partial^2 |G\rangle}{\partial \pi^i \partial \pi^k}.$$
 (5.13)

Now one should build the qubit spinor in the local basis ( $|N\rangle, |D\rangle$ ) for the quantum question in respect with the measurement of some local dynamical variable  $\vec{D}$  at some GCS which may be marked by the normal  $|N\rangle$ . We will assume that there is *natural state*  $|\widetilde{D}\rangle$  of the quantum system in the LRF representation equal to the renormalized lift of LDV  $\vec{D} \in T_\pi CP(N-1)$  into the environmental Hilbert space  $\mathcal{H}$ , and there is *expectation state*  $\hat{D}|D_{expect}\rangle = \lambda_D |D_{expect}\rangle$ , associated with the tuning of “measuring device”. This notional measuring device is associate with the local unitary projector along the normal  $|N\rangle$  onto the natural state  $|\widetilde{D}\rangle$ . In fact it defines the covariant derivative in  $CP(N-1)$ . The lift-vectors  $|N\rangle, |D\rangle$  are given by the solutions of (5.12) arising under interaction used for the measurement of the LDV  $\vec{D}$ . In general  $|D\rangle$  it is not a tangent vector to  $CP(N-1)$ . But renormalized vector defined as the covariant derivative  $|\widetilde{D}\rangle = |D\rangle - \langle Norm|D\rangle |Norm\rangle$  is a tangent vector to  $CP(N-1)$  if  $|Norm\rangle = \frac{|N\rangle}{\sqrt{\langle N|N\rangle}}$ . The operation of the  $|\widetilde{D}\rangle$  renormalization is the orthogonal (unitary) projector. Indeed,

$$\begin{aligned}|\widetilde{D}\rangle &= (|D\rangle - \langle Norm|D\rangle |Norm\rangle) \\ &= |D\rangle - \langle Norm|D\rangle |Norm\rangle \\ &- \langle Norm|(|D\rangle - \langle Norm|D\rangle |Norm\rangle)|Norm\rangle \\ &= |D\rangle - \langle Norm|D\rangle |Norm\rangle = |\widetilde{D}\rangle.\end{aligned}\quad (5.14)$$

Then at the point  $(\pi^1, \dots, \pi^{N-1})$  one has two components of the qubit spinor

$$\begin{aligned}\alpha_{(\pi^1, \dots, \pi^{N-1})} &= \frac{\langle N|D_{expect}\rangle}{\langle N|N\rangle} \\ \beta_{(\pi^1, \dots, \pi^{N-1})} &= \frac{\langle \widetilde{D}|D_{expect}\rangle}{\langle \widetilde{D}|\widetilde{D}\rangle}\end{aligned}\quad (5.15)$$

then at the infinitesimally close point  $(\pi^1 + \delta^1, \dots, \pi^{N-1} + \delta^{N-1})$  one has new qubit spinor

$$\alpha_{(\pi^1 + \delta^1, \dots, \pi^{N-1} + \delta^{N-1})} = \frac{\langle N'|D_{expect}\rangle}{\langle N'|N'\rangle}$$

$$\beta_{(\pi^1+\delta^1, \dots, \pi^{N-1}+\delta^{N-1})} = \frac{\langle \widetilde{D}' | D_{expect} \rangle}{\langle \widetilde{D}' | \widetilde{D}' \rangle} \quad (5.16)$$

where the basis  $(|N' \rangle, |\widetilde{D}' \rangle)$  is the lift of the parallel transported  $(|N \rangle, |\widetilde{D} \rangle)$  from the infinitesimally close  $(\pi^1 + \delta^1, \dots, \pi^{N-1} + \delta^{N-1})$  back to  $(\pi^1, \dots, \pi^{N-1})$ .

These two infinitesimally close qubit spinors being expressed as functions of  $\theta, \phi, \psi, R$  and  $\theta + \epsilon_1, \phi + \epsilon_2, \psi + \epsilon_3, R + \epsilon_4$ , represented as follows

$$\eta = R \begin{pmatrix} \cos \frac{\theta}{2} (\cos \frac{\phi_1 - \psi_1}{2} - i \sin \frac{\phi_1 - \psi_1}{2}) \\ \sin \frac{\theta}{2} (\cos \frac{\phi_1 + \psi_1}{2} + i \sin \frac{\phi_1 + \psi_1}{2}) \end{pmatrix} = R \begin{pmatrix} C(c - is) \\ S(c_1 + is_1) \end{pmatrix} \quad (5.17)$$

and

$$\begin{aligned} \eta + \delta\eta &= R \begin{pmatrix} C(c - is) \\ S(c_1 + is_1) \end{pmatrix} \\ + R \begin{pmatrix} S(is - c)\epsilon_1 - C(s + ic)\epsilon_2 + C(s + ic)\epsilon_3 + C(c - is)\frac{\epsilon_4}{R} \\ C(c_1 + is_1)\epsilon_1 + S(ic_1 - s_1)\epsilon_2 - S(s_1 - ic_1)\epsilon_3 + S(c_1 + is_1)\frac{\epsilon_4}{R} \end{pmatrix} \end{aligned} \quad (5.18)$$

may be connected with infinitesimal ‘‘Lorentz spin transformations matrix’’ [14]

$$L = \begin{pmatrix} 1 - \frac{i}{2}\tau(\omega_3 + ia_3) & -\frac{i}{2}\tau(\omega_1 + ia_1 - i(\omega_2 + ia_2)) \\ -\frac{i}{2}\tau(\omega_1 + ia_1 + i(\omega_2 + ia_2)) & 1 - \frac{i}{2}\tau(-\omega_3 - ia_3) \end{pmatrix}. \quad (5.19)$$

Then accelerations  $a_1, a_2, a_3$  and angle velocities  $\omega_1, \omega_2, \omega_3$  may be found in the linear approximation from the equation

$$\eta + \delta\eta = L\eta \quad (5.20)$$

as functions of the qubit spinors components depending on local coordinates  $(\pi^1, \dots, \pi^{N-1})$ .

Hence the infinitesimal Lorentz transformations define small ‘‘space-time’’ coordinates variations. It is convenient to take Lorentz transformations in the following form  $ct' = ct + (\vec{x}\vec{a})d\tau$ ,  $\vec{x}' = \vec{x} + ct\vec{a}d\tau + (\vec{\omega} \times \vec{x})d\tau$ , where I put  $\vec{a} = (a_1/c, a_2/c, a_3/c)$ ,  $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$  [14] in order to have for  $\tau$  the physical dimension of time. The coordinates  $x^\mu$  of points in this space-time serve in fact merely for the parametrization of deformations of the ‘‘field shell’’ arising under its motion according to non-linear field equations [2, 3].

## 6 Field shell equations (FSE)

In order to find the “field shell” of the perturbed GCS one should establish some wave equations in the dynamical space-time. All these notions require more precise definitions. Namely, say, in the simplest case of  $CP(1)$ , the “field shells” being represented in the spherical coordinates are the classical vector fields  $\Omega^\alpha = \frac{x^\alpha}{r}(\omega + i\gamma)$ ,  $1 \leq \alpha \leq 3$  giving the temps of the GCS variations. The tensor fields  $1 \leq \alpha \leq 8, 15, \dots, N^2 - 1$  will be discussed elsewhere. Note, that the maximal number of EQS  $a = 0, 1, \dots, N, \dots$  now strongly connected with the tensor character of the GCS driving field  $\Omega^\alpha$ . These fields are “classical” since they are not subjected to quantization directly, i.e. by the attribution of the fermionic or bosonic commutation relations. They obey to nonlinear field equations. Their internal dynamical variables like spin, charge, etc., will be a consequence of their dynamical structure.

“Particle” now associated with the “field shell” in the dynamical space-time (see below), given locally by the Hamiltonian vector field  $\vec{H}$ . At each point  $(\pi^1, \dots, \pi^{N-1})$  of the  $CP(N-1)$  one has an “expectation value” of the  $\vec{H}$  defined by a measuring device. But displaced GCS may be reached along of one of continuum pathes. Therefore the comparison of two vector fields and their “expectation values” in neighborhood points requires some natural rule. The “natural” in our case means that the comparison has sense only for same “particle” or for its “field shell”. For this reason one should have a “self-identification” procedure. The affine parallel transport in  $CP(N-1)$  of vector fields is a natural and the simplest rule for the comparison of corresponding “field shells”. Physically the self-identification of “particle” literally means that its Hamiltonian vector field is the Fubini-Study covariant constant.

But there are questions: what should coincide, and what is the “expected” and what is “the detected particles”, because we have not particles at all? Since we have only the unitary fields  $\Omega^\alpha$  as parameters of GCS transformations we assume that in accordance with the super-equivalence principle under the infinitesimal shift of the unitary field  $\delta\Omega^\alpha$  in the dynamical space-time, the shifted Hamiltonian field should coincide with the infinitesimal shift of tangent Hamiltonian field generated by the parallel transport in  $CP(N-1)$  during NSHP time  $\delta\tau$  [18]. Thus one has

$$\hbar(\Omega^\alpha + \delta\Omega^\alpha)\Phi_\alpha^k = \hbar\Omega^\alpha(\Phi_\alpha^k - \Gamma_{mn}^k \Phi_\alpha^m V^n \delta\tau) \quad (6.1)$$

and, hence,

$$\frac{\delta\Omega^\alpha}{\delta\tau} = -\Omega^\alpha \Gamma_{mn}^m V^n \quad (6.2)$$



We introduce the dynamical space-time coordinates  $x^\mu$  as state-dependent quantities, transforming in accordance with the local Lorentz transformations  $x^\mu + \delta x^\mu = (\delta^\mu_\nu + \Lambda^\mu_\nu \delta\tau) x^\nu$ . The parameters of  $\Lambda^\mu_\nu(\pi^1, \dots, \pi^{N-1})$  depend on the local transformations of LRF in  $CP(N-1)$  described in the previous paragraph. Assuming a spherically symmetrical solution, we will use the coordinates ( $x^0 = ct, x^1 = r \sin \Theta \cos \Phi, x^2 = r \sin \Theta \sin \Phi, x^3 = r \cos \Theta$ ). In the case of spherical symmetry,  $\Omega^1 = (\omega + i\gamma) \sin \Theta \cos \Phi, \Omega^2 = (\omega + i\gamma) \sin \Theta \sin \Phi, \Omega^3 = (\omega + i\gamma) \cos \Theta$  and in the general case of the separability of the angle and radial parts, one has  $\Omega^\alpha = \sum C_{l,m}^\alpha Y_{l,m}(\Theta, \Phi)(\omega + i\gamma) = \sum C_{l,m}^\alpha Y_{l,m}(\Theta, \Phi)\Omega$ . Then taking into account the expressions for the “4-velocity”  $v^\mu = \frac{\delta x^\mu}{\delta \tau} = \Lambda^\mu_\nu(\pi^1, \dots, \pi^{N-1})x^\nu$  one has the field equation

$$v^\mu \frac{\partial \Omega}{\partial x^\mu} = -\Omega \Gamma_{mn}^m V^n, \quad (6.3)$$

where

$$\begin{aligned} v^0 &= (\vec{x}\vec{a}) \\ \vec{v} &= ct\vec{a} + (\vec{\omega} \times \vec{x}). \end{aligned} \quad (6.4)$$

If one wishes to find the field corresponding to a given trajectory, say, a geodesic in  $CP(N-1)$ , then, taking into account that any geodesic as whole belongs to some  $CP(1)$ , one may put  $\pi^1 = e^{i\phi} \tan(\sigma\tau)$ . Then  $V^1 = \frac{d\pi^1}{d\tau} = \sigma \sec^2(\sigma\tau) e^{i\phi}$ , and one has a linear wave equations for the gauge unitary field  $\Omega^\alpha$  in the dynamical space-time with complicated coefficient functions of the local coordinates  $(\pi^1, \dots, \pi^{N-1})$ . Under the assumption  $\tau = wt$  this equation has following solution

$$\omega + i\gamma = (F_1(r^2 - c^2t^2) + iF_2(r^2 - c^2t^2)) \exp\left(-2wc \int_0^t dp \frac{\tan(wp)}{A\sqrt{c^2(p^2 - t^2) + r^2}}\right), \quad (6.5)$$

where  $F_1, F_2$  are an arbitrary function of the interval  $s^2 = r^2 - c^2t^2$ ,  $(\vec{a}, \vec{x}) = Ar \cos(\chi)$ ,  $A = \sqrt{a_x^2 + a_y^2 + a_z^2}$  and  $r = \sqrt{x^2 + y^2 + z^2}$ . The angle  $\chi$  in fact is defined by a solution of the equation (5.20). I used  $\chi = \pi$  since for us now interesting only “radial boost turned toward the center of the field shell”.

The general factor demonstrates the diffusion of the light cone (mass shell) due to the boosts. Thus our results consist with the so-called “off-shell” idea of Horwitz-Piron-Stueckelberg [25].

## 7 Quasi-Hamiltonian equations

The theory of the quasi-linear PDE field equations (6.3) is well known [26, 27]. I wish apply general approach looking on the quasi-linear PDE as a particular case of the

nonlinear PDE. Let me write such equation in the form

$$G(x^\mu, P_\mu, \Omega) = v^\mu \frac{\partial \Omega}{\partial x^\mu} + \Omega \Gamma_{mn}^m V^n = 0, \quad (7.1)$$

where I put  $P_\mu = \frac{\partial \Omega}{\partial x^\mu}$ . Such PDE equations demonstrate a natural “wave-corpustular duality” in the following sense. Equation (7.1) itself is field equation relative  $\Omega$ . Then “4-velocity”  $v^\mu = \frac{dx^\mu}{d\tau}$  against the parameter of evolution  $\tau$  may be treated as velocities of “corpustules” moving along trajectories in the dynamical space-time. We can see it from the following calculations. The equation (7.1) define the hyper-surface  $E$  in the 9-dimension space of 1-jets [27]. The phase curves lie as whole on  $E$  and obey following quasi-Hamiltonian system.

$$\frac{dG(x^\mu, P_\mu, \Omega)}{d\tau} = G_{x^\mu} v^\mu + G_{P_\mu} \frac{dP_\mu}{d\tau} + G_\Omega \frac{d\Omega}{d\tau} = 0, \quad (7.2)$$

Using the explicit form of  $G$ , one can rewrite this equation as follows:

$$G_{x^\mu} v^\mu + v^\mu \frac{dP_\mu}{d\tau} + G_\Omega \frac{\partial \Omega}{\partial x^\mu} v^\mu = 0. \quad (7.3)$$

Then the full characteristic system reads now

$$\begin{aligned} \frac{dx^\mu}{d\tau} &= \frac{\partial G}{\partial P_\mu} = v^\mu, \\ \frac{dP_\mu}{d\tau} &= -\frac{\partial G}{\partial x^\mu} - \frac{\partial G}{\partial \Omega} P_\mu, \\ \frac{d\Omega}{d\tau} &= \frac{\partial \Omega}{\partial x^\mu} v^\mu = \frac{\partial \Omega}{\partial x^\mu} \frac{\partial G}{\partial P_\mu} = P_\mu \frac{\partial G}{\partial P_\mu}. \end{aligned} \quad (7.4)$$

Here one has generalized Hamiltonian equations describing a “corpustular” point-wise motion in the 4D dynamical space-time. The analysis of stability of their solutions and the physical sense of their Schrödinger’s quantization require future investigations.

## 8 Conclusion

The main new points of my approach are following:

A. I use the notion of “elementary quantum motions” (EQM)  $|\hbar a >$  with well defined quantized Planck’s action  $S_a = \hbar a$  instead of the notion of “elementary particles”. Their GCS’s serve as an abstract formalization of the “quasi-classical” description of a quantum setup or “Schrödinger’s lump” [1].

B. The quantum phase space  $CP(N - 1)$  serves as the base of the tangent fibre bundle of the local dynamical variables. The special cross-section of this bundle and affine gauge field are geometric tools for the quantum measurement in the state-dependent dynamical space-time.

C. Integration over all pathes (alternatives) realizes the objective approach in quantum theory. The dominate contribution will be given by the geodesic of  $CP(N - 1)$  spanning two GCS's [5].

The technical details are as follows:

1. The projective representation of pure  $N$ -dimension quantum states (one could think of arbitrary large  $N$ ), provides a natural non-linear realization of the  $G = SU(N)$  group manifold and the coset sub-manifold  $G/H = SU(N)/S[U(1) \times U(N - 1)] = CP(N - 1)$ . I consider the generators of this group as LDV's [5] of the model.

2. These quantum dynamical variables are represented by the tangent vector fields to  $CP(N - 1)$ . Embedding of  $CP(N - 1)$  into  $\mathcal{H} = C^N$  provides the measurement procedure for the dynamical variables.

3. Quantum measurement “creates” local dynamical space-time capable of detecting the coincidence of expectation and measured values of these quantum dynamical variables.

4. The affine parallel transport, associated with the Fubini-Study metric, accompanied with “Lorentz spin transformation matrix” [14], establish this coincidence due to the identification of the parallel transported LDV at different GCS's.

5. The parametrization of the measurement results with the help of *attributed* local space-time coordinates is in fact the embedding of quantum dynamics in Hilbert space into 4D world. This procedure is well definite due to the existence of the infinitesimal  $SL(2, C)$  transformations of the qubit spinor treated as Lorentz transformations of local space-time coordinates.

6. The quasi-linear PDE for non-Abelian gauge field in dynamical space-time naturally related to the ODE of their characteristics. The last ones are similar to the Hamilton canonical equations. Their quantization leads to Schroödinger-like equations whose properties will be discussed elsewhere.

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